# Many for one rank

## Calibration-free, accelerated dynamic MRI based on rank-one matrix recovery

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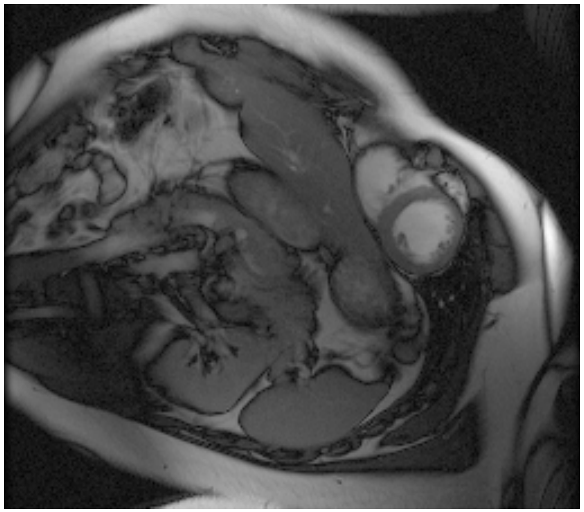
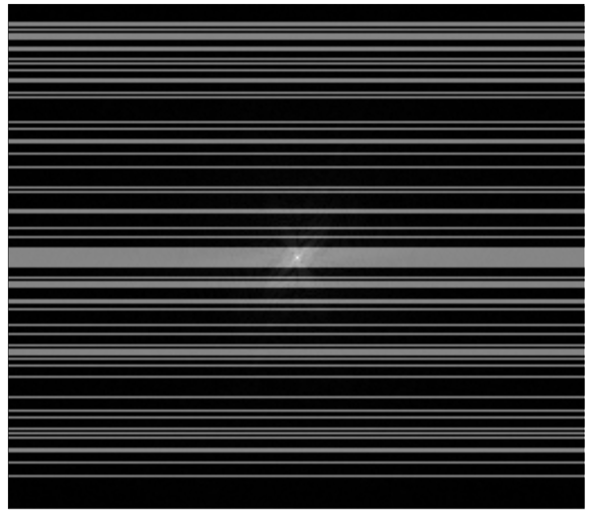
# Outline

- Motivation
  - Compressed sampling for accelerated MRI
  - Parallel, multichannel imaging
  - Dynamics
- Blind deconvolution
  - Methods for fully sampled data
- Dynamic MRI + blind deconvolution
  - Joint channel and image estimation as rank-one recovery
  - Experimental results

# Motivation: Accelerated MRI



**Dynamic MRI**
Sequence of images with similarities across space and time



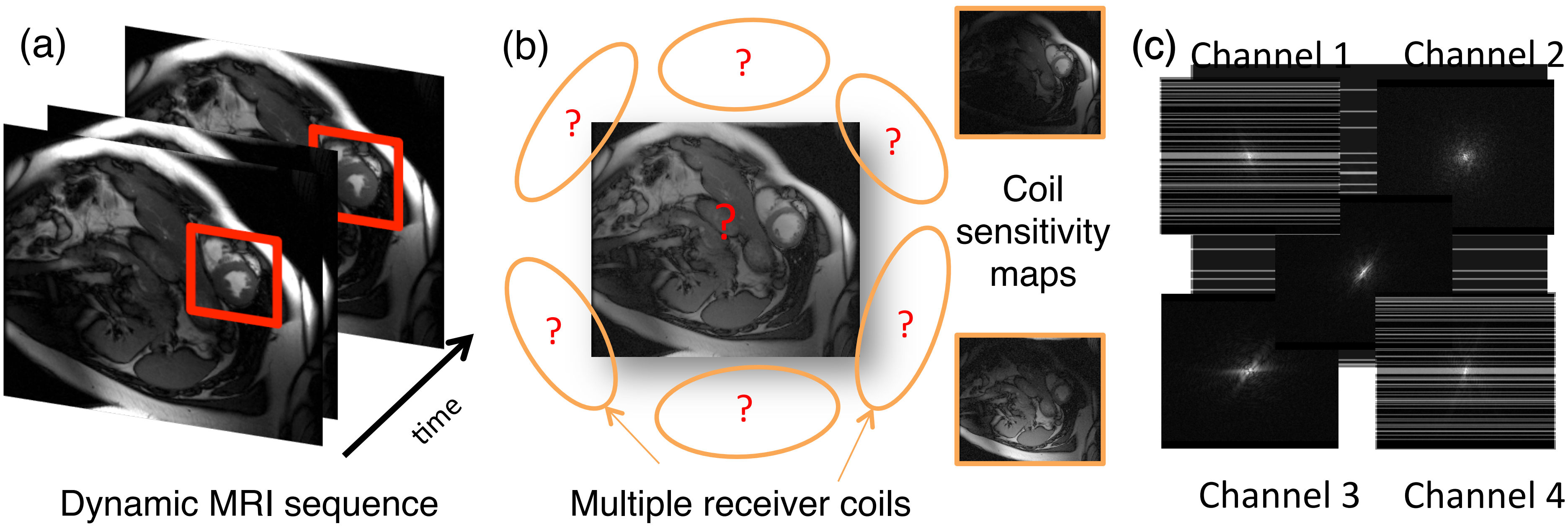
**Compressed sampling**
Subsample k-space to accelerate data acquisition

**Naïve reconstruction**
(Zero-filled inverse FFT)
Spatial aliasing due to undersampling in k-space

**Obviously, reconstruction with sparse and low-dimensional signal models that exploit spatial and temporal structures will provide better reconstruction**
[Lustig et al., *Sparse MRI*, MRM, 2007]

# Problem description

- **Calibration-free:** Jointly estimate MR image sequence and coil sensitivity profiles from under-sampled data




**sub-sampled k-space**

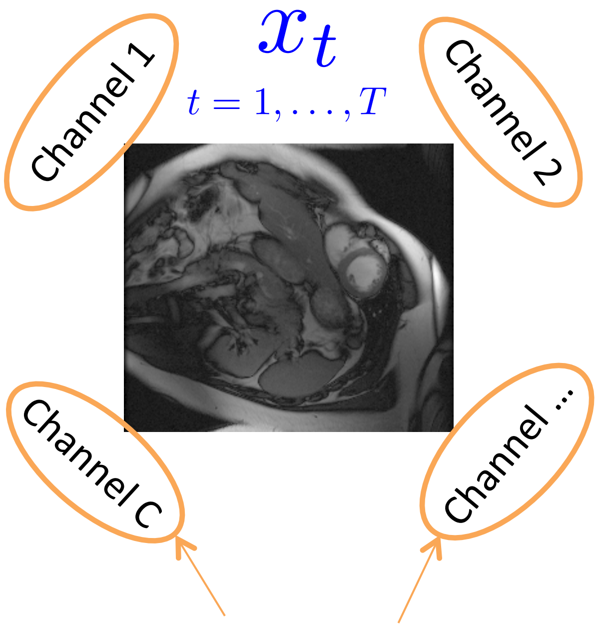
Modulated images down-sampled in k-space

$$\mathbf{y}_t^c = R_{\Omega_t}\mathcal{F}(\mathbf{s}_c \odot \mathbf{x}_t)$$

Partially observed Fourier samples

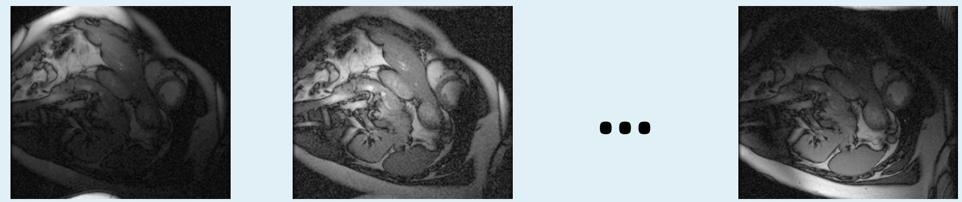
MR images modulated by coil sensitivities

# Multichannel dynamic MRI



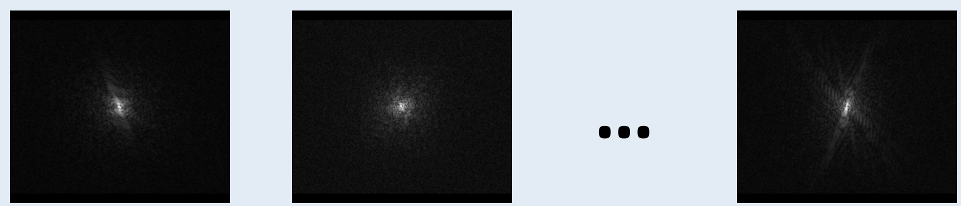

**Parallel MRI**

Coil/channel



$s_1 \odot x_t$ $s_2 \odot x_t$ ... $s_C \odot x_t$

**Images modulated by coil sensitivities in spatial domain**



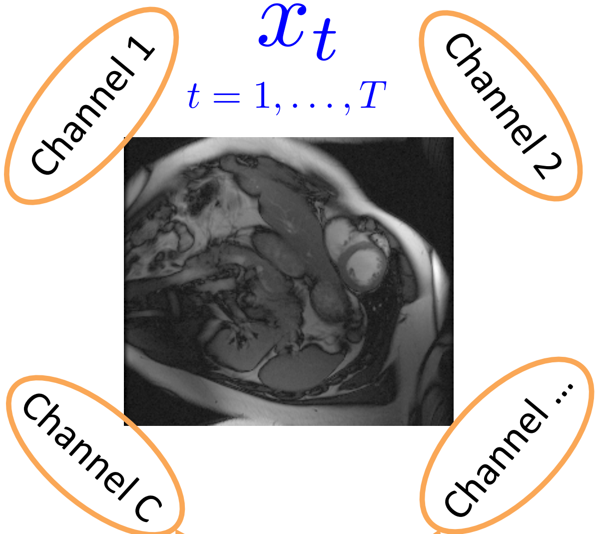
$\mathcal{F}(s_1 \odot x_t)$ $\mathcal{F}(s_2 \odot x_t)$ ... $\mathcal{F}(s_C \odot x_t)$

**Fourier domain representation**

# Multichannel dynamic MRI

$x_t$

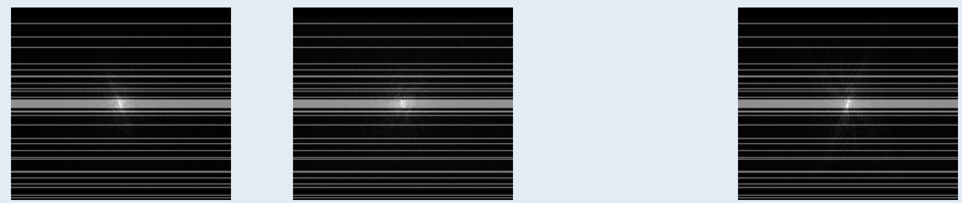
$t = 1, \ldots, T$




Multiple receiver coils

**Parallel MRI**

Coil/channel



$\mathcal{R}_{\Omega_t}\mathcal{F}(s_1 \odot x_t)$ $\mathcal{R}_{\Omega_t}\mathcal{F}(s_C \odot x_t)$

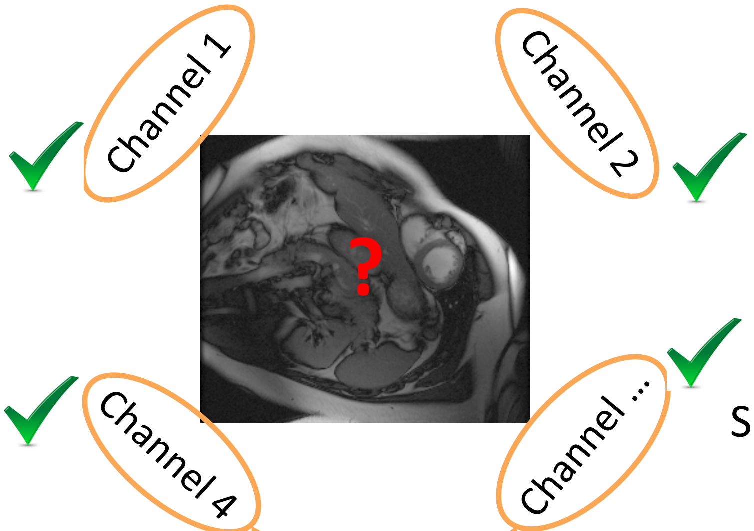
**Subsampling in Fourier domain (k-space)**

$$y_t^c = R_{\Omega_t}\mathcal{F}(s_c \odot x_t)$$

Subsampled k-space

MR images modulated by coil sensitivities

# Multichannel dynamic MRI




Multiple receiver coils

**Parallel MRI**

$$y_t^c = R_{\Omega_t}\mathcal{F}(s_c \odot x_t)$$

Subsampled k-space

MR images modulated by coil sensitivities

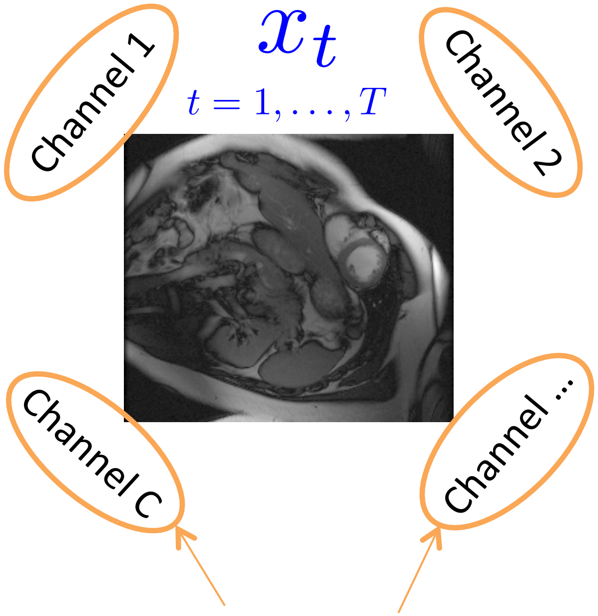
**Known coil sensitivity responses**

**Unknown dynamic MRI sequence**

$$y_t^c = R_{\Omega_t}\mathcal{F}\Sigma_{s_c} x_t$$

A system of **linear** equations *!!!*

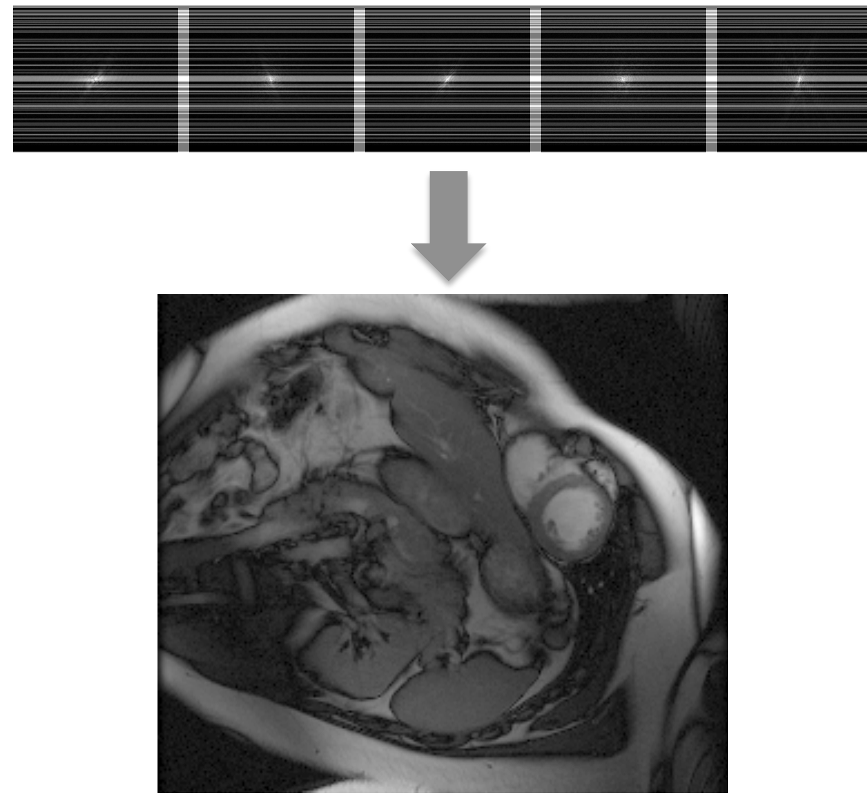
# Multichannel dynamic MRI




Multiple receiver coils

**Parallel MRI**

**4x subsampled k-space**

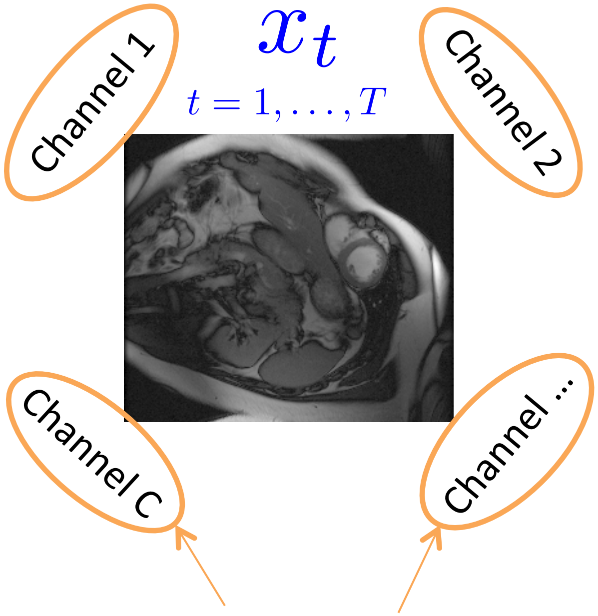


**Frame-by-frame reconstruction**
Oversampling in space due to redundant coils helps with the reconstruction

$$y_t^c = R_{\Omega_t} \mathcal{F}(s_c \odot x_t)$$

**Stack all measurements into a linear system** [Pruessmann et al. SENSE, MRM, 1999]

# Multichannel dynamic MRI



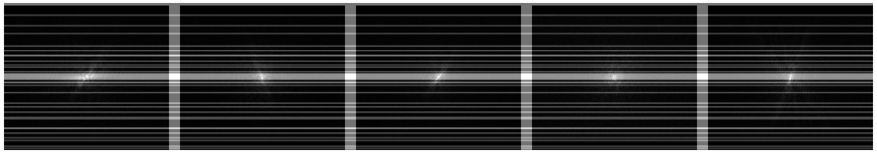

Multiple receiver coils

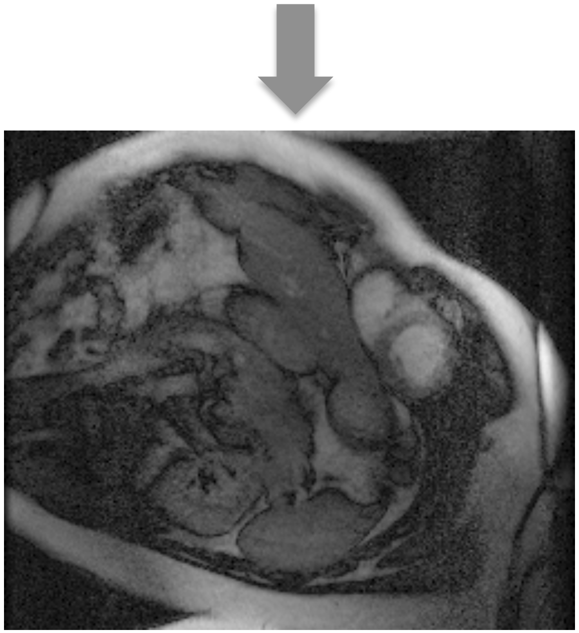
**Parallel MRI**

$$y_t^c = R_{\Omega_t} \mathcal{F}(s_c \odot x_t)$$

**Stack all measurements into a linear system** [Pruessmann et al. SENSE, MRM, 1999]

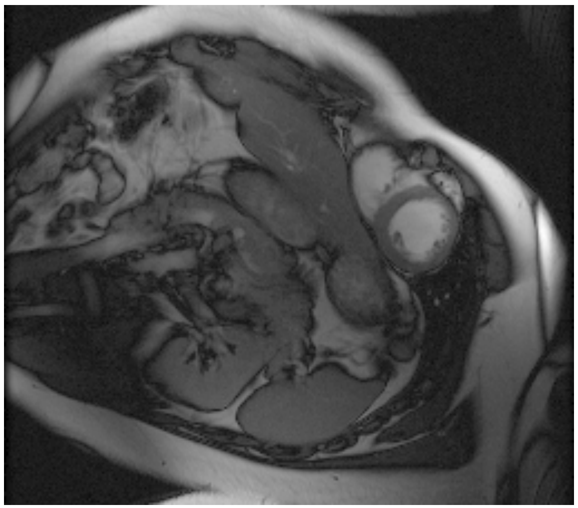
**8x** subsampled k-space



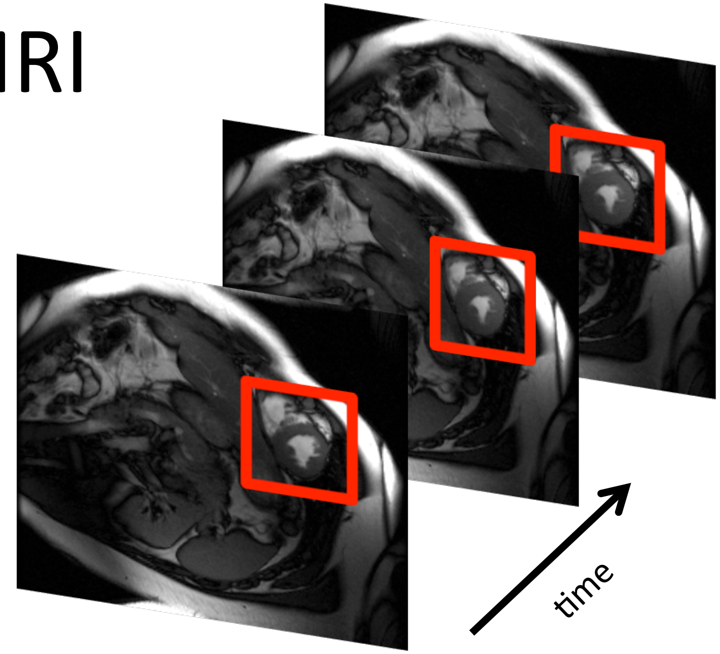


**Frame-by-frame reconstruction**
Multiple coils/channels provide redundancy to offset small subsampling factors only

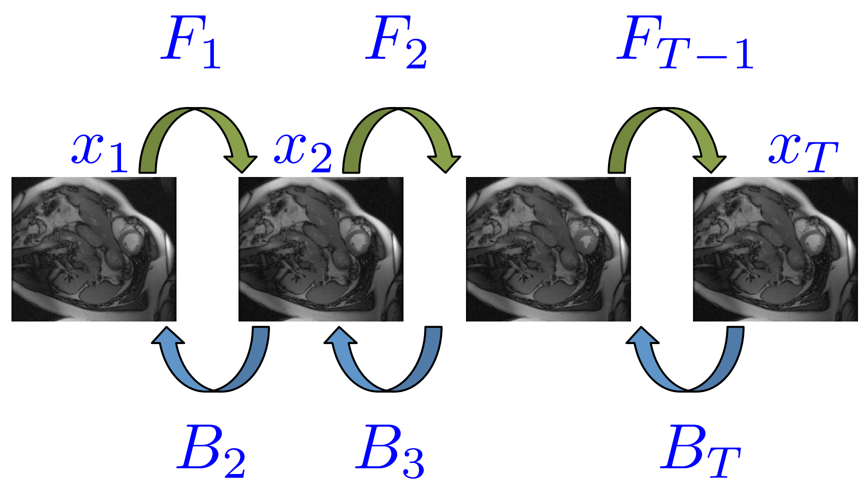
# Multichannel **dynamic** MRI



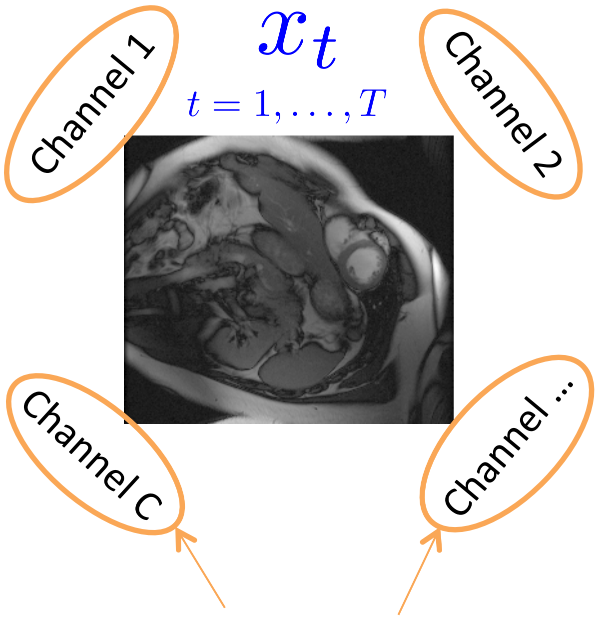
**Dynamic MRI**
A video sequence with similarities across space and time




*Small region with significant action*
*Spatial and temporal redundancies (like videos)*



# Multichannel dynamic MRI




Multiple receiver coils

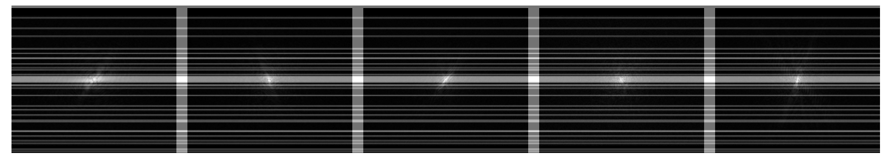
**Parallel MRI**

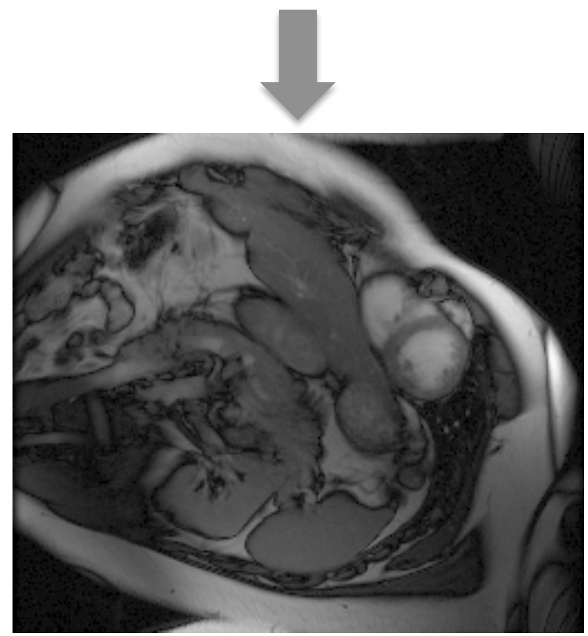
$$y_t^c = R_{\Omega_t} \mathcal{F}(s_c \odot x_t)$$

$$x_{t+1} \approx F_t x_t$$

**Motion-adaptive constraint**

**8x subsampled k-space**



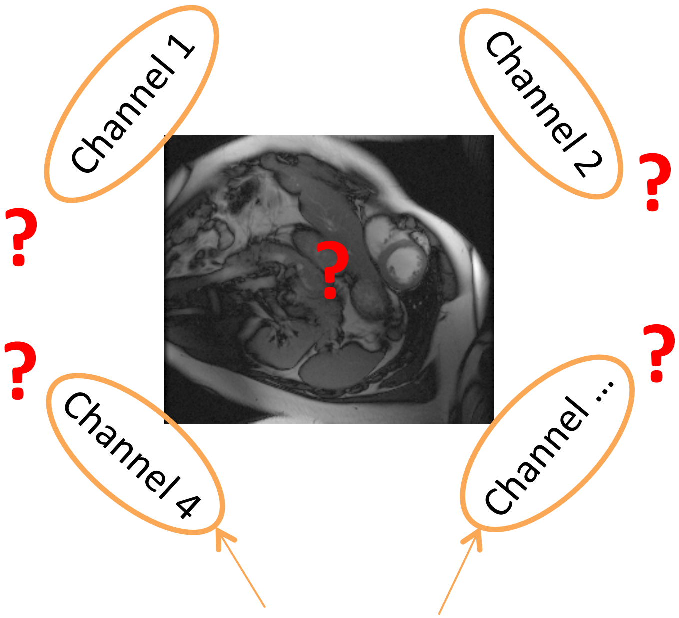


**Motion-adaptive reconstruction**

Exploit temporal structure in the reconstruction

[Jung et al., k-t FOCUSS, MRM, 2009; Asif et al., MASTeR, MRM, 2013]

# Autocalibration?




**Parallel MRI**

$$y_t^c = R_{\Omega_t}\mathcal{F}(s_c \odot x_t)$$

Subsampled k-space

MR images modulated by coil sensitivities

**Unknown coil sensitivity responses**
**Unknown dynamic MRI sequence**

$$y_t^c = R_{\Omega_t}\mathcal{F}\Sigma_{s_c} x_t$$

A system of **bilinear** equations *!!!*

# Blind deconvolution: fully sampled data

- A common technique in blind deconvolution schemes is to exploit *cross-relation consistency*:

**Consider a multi-channel system**

$$y_i = x \circledast h_i \quad \text{for } i = 1, 2, \ldots$$

**Cross-relation consistency:**

$$y_i \circledast h_j = y_j \circledast h_i \quad \text{for all} \quad i, j$$

**The null-space of the matrix defined by these equations provides all the feasible solutions**

[Tong et al., 1991; Harikumar & Bresler, 1999; … ]

# MRI autocalibration: fully sampled regions

- Conventional coil estimation methods use a small, fully sampled k-space region

**Learn filters from autocalibration data**

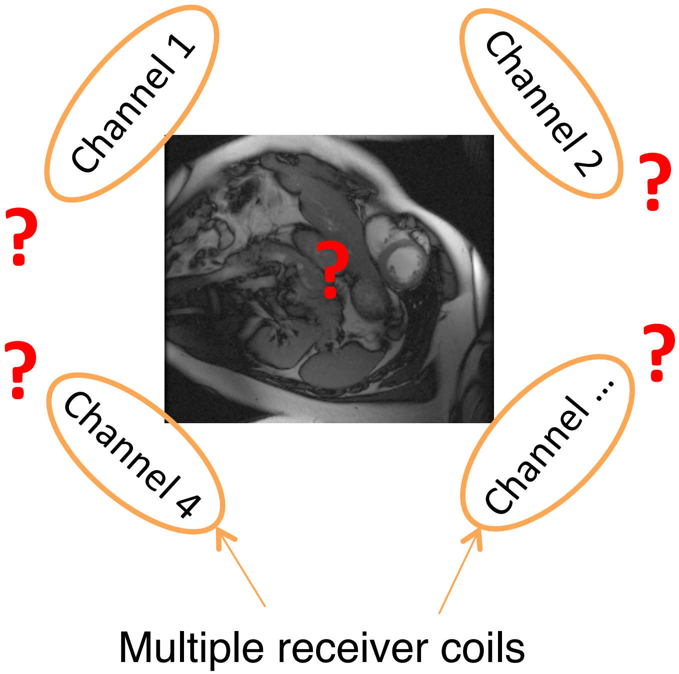
$$y_c = \sum_k g_{c,k} \circledast y_k$$

compute a filter for local regions that best describes k-space samples using all its neighboring pixels

- With the exception of SAKE, all these methods require a small, fully sampled region in the k-space

[Sodickson et al., *SMASH*, MRM, 1997; Griswold et al., *GRAPPA*, MRM, 2002; Lustig et al., *SPIRIT*, MRM, 2010; Uecker et al., *ESPIRIT*, MRM, 2014; Shin et al., *SAKE*, MRM, 2014]

# Autocalibration?

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$$y_t^c = R_{\Omega_t} \mathcal{F}(s_c \odot x_t)$$

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$$y_t^c = R_{\Omega_t} \mathcal{F} \Sigma_{s_c} x_t$$

A system of **bilinear** equations *!!!*

# Blind deconvolution + dynamic MRI

**Stack all the measurements into a large system:**

$$y_t^c = R_{\Omega_t}\mathcal{F}(s_c \odot x_t)\Big|_{\substack{c=1,\ldots,C\\ t=1,\ldots,T}} \longrightarrow y = \mathcal{A}(xs^*)$$

**Sample diagonal entries of the rank-1 matrix:** $xs^*$

$$xs^* = \begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix} \begin{bmatrix} {s_1}^* & \ldots & {s_C}^* \end{bmatrix} = \begin{bmatrix} x_1{s_1}^* & \cdots & x_1{s_C}^* \\ \vdots & \ddots & \vdots \\ x_T{s_1}^* & \cdots & x_T{s_C}^* \end{bmatrix}$$

# Blind deconvolution + dynamic MRI

**Solve a nonlinear problem for rank-1 recovery**

$$\underset{x,s}{\text{minimize}} \; \|x\|_2^2 + \|s\|_2^2 \;\; \text{s.t.} \;\; y = \mathcal{A}(xs^*)$$

Low-rank factorization for matrix completion/reconstruction
[Recht et al., 2010; Burer & Monteiro, 2005; Jain et al., 2013; Cai et al., 2010; …]

**Add motion-adaptive prior:**

$$\underset{x,s}{\text{minimize}} \; \|x\|_2^2 + \|s\|_2^2 + \sigma\|y - \mathcal{A}(xs^*)\|_2^2$$

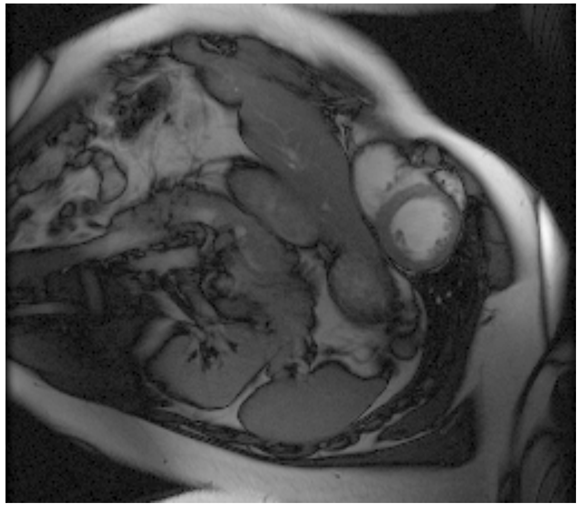
$$+\lambda \sum_{t=1}^{T-1} \|x_{t+1} - F_t x_t\|_2^2$$

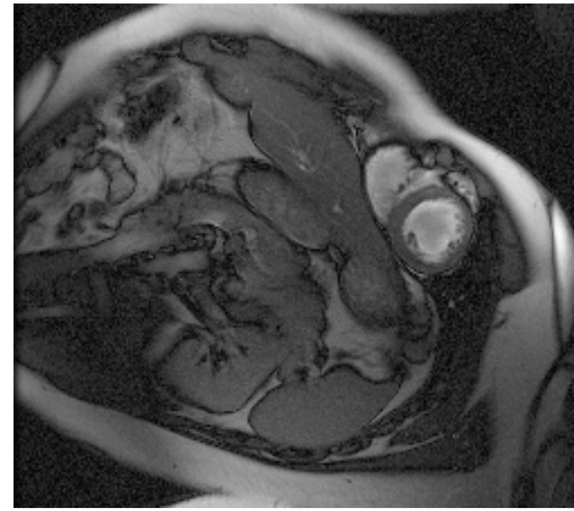
# Experimental results

**Known sensitivity maps** **Unknown sensitivity maps**

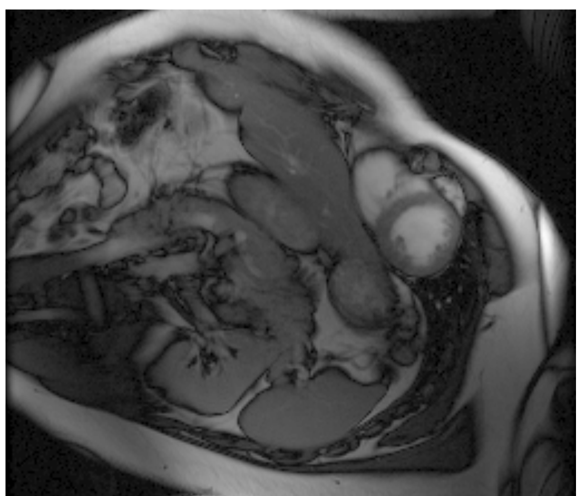
**R=4**

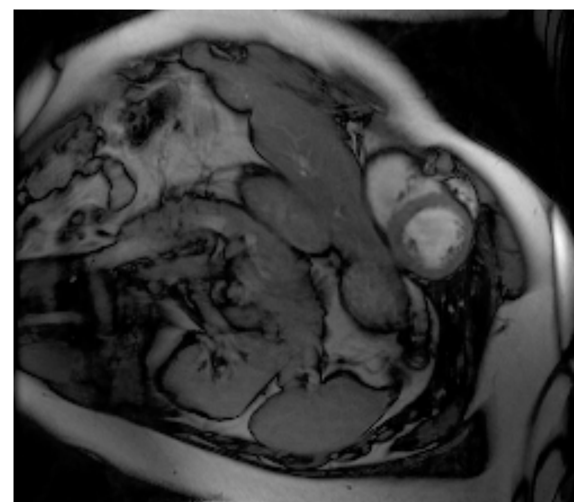
**Without motion**





**With motion**



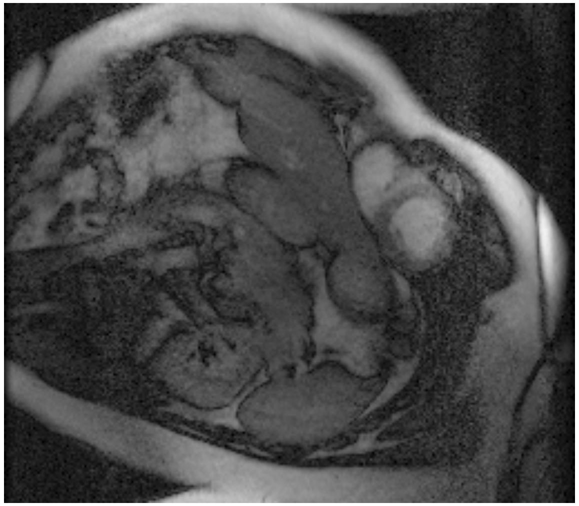


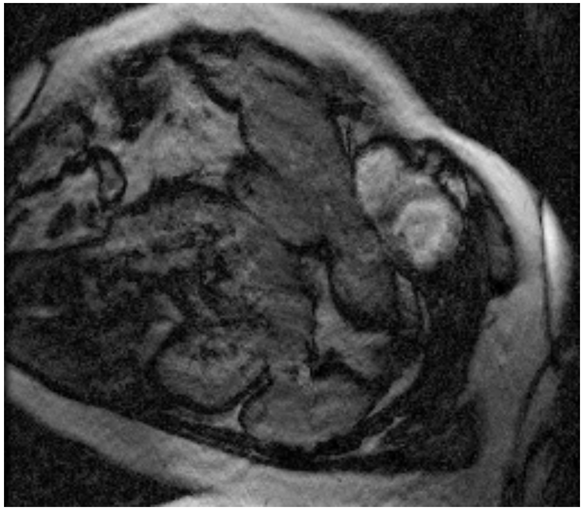
# Experimental results

**Known sensitivity maps** **Unknown sensitivity maps**

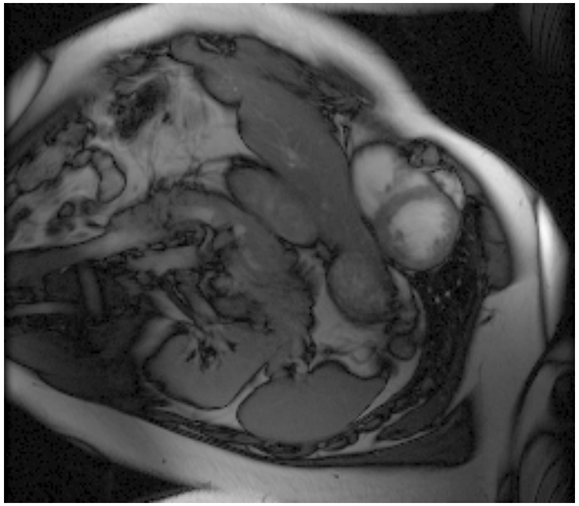
**R=8**

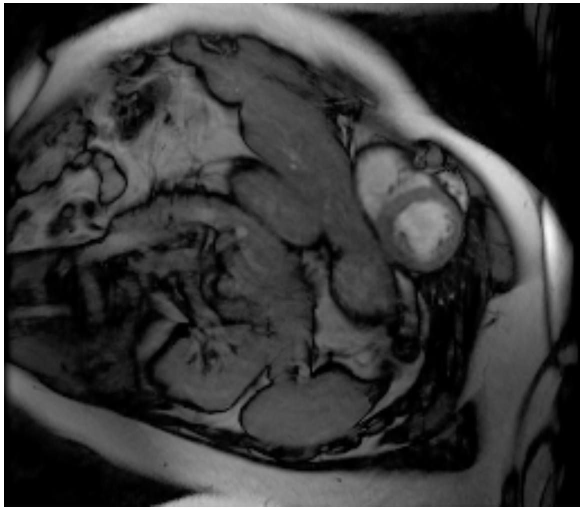
**Without motion**





**With motion**



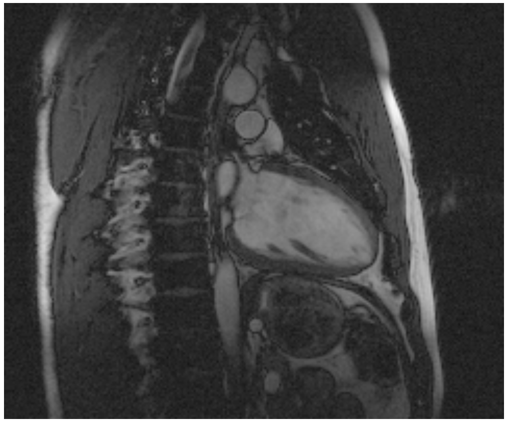


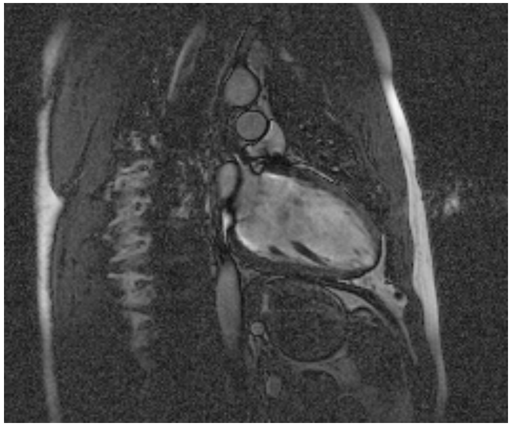
# Experimental results

**Known sensitivity maps** | **Unknown sensitivity maps**

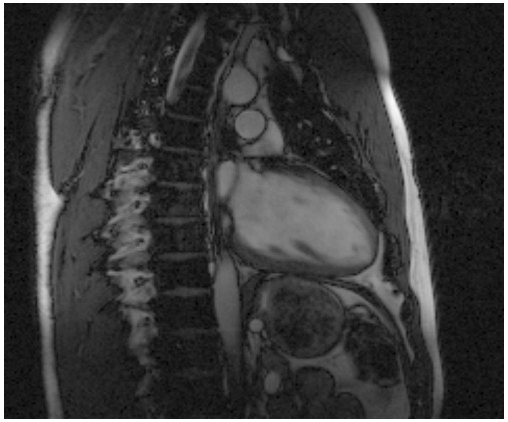
**R=4**

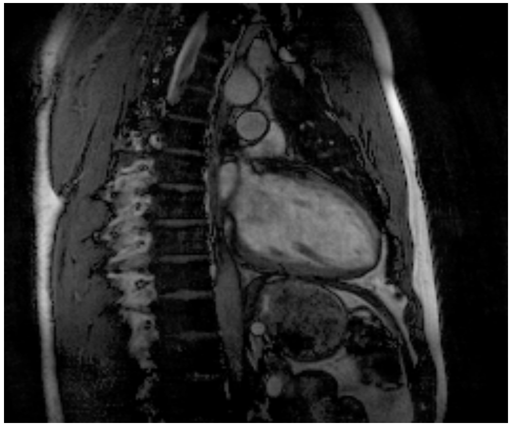
**Without motion**



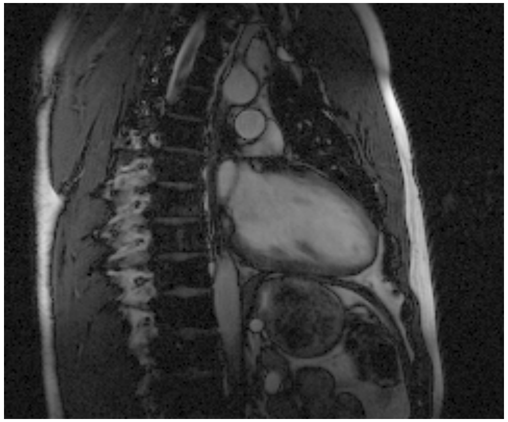
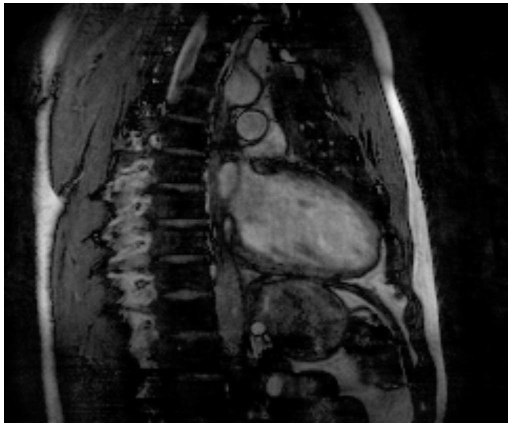


**With motion**



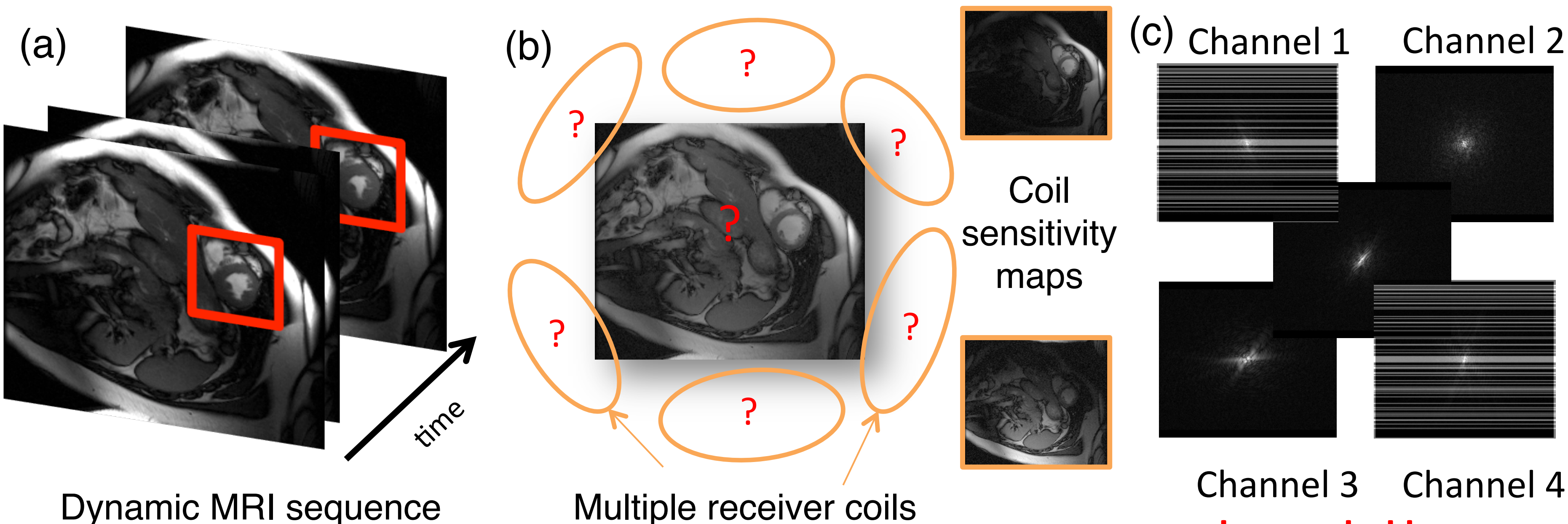


# Experimental results

| | **Known sensitivity maps** | **Unknown sensitivity maps** |
|---|---|---|
| **R=8** | | |
| **Without motion** |  |  |
| **With motion** |  |  |

# Summary

- **Calibration-free:** Jointly estimate MR image sequence and coil sensitivity profiles from under-sampled data




**sub-sampled k-space**

Modulated images down-sampled in k-space

$$\mathbf{y}_t^c = R_{\Omega_t}\mathcal{F}(\mathbf{s}_c \odot \mathbf{x}_t)$$

Partially observed Fourier samples

MR images modulated by coil sensitivities

# References


- M. Asif, L. Hamilton, M. Brummer, J. Romberg, *Motion-adaptive spatio-temporal regularization (MASTeR) for accelerated dynamic MRI*, Magnetic Resonance in Medicine, September 2013
- M. Asif, F. Fernandes, J. Romberg, *Low-complexity video compression and compressive sensing,* Asilomar 2013
- A. Ahmed, B. Recht, J. Romberg, *Blind deconvolution using convex programming.* IEEE Trans. Info. Theory, March 2014.


**Data and codes: http://users.ece.gatech.edu/sasif/dynamicMRI**

**http://dsp.rice.edu/**

# Backup slides

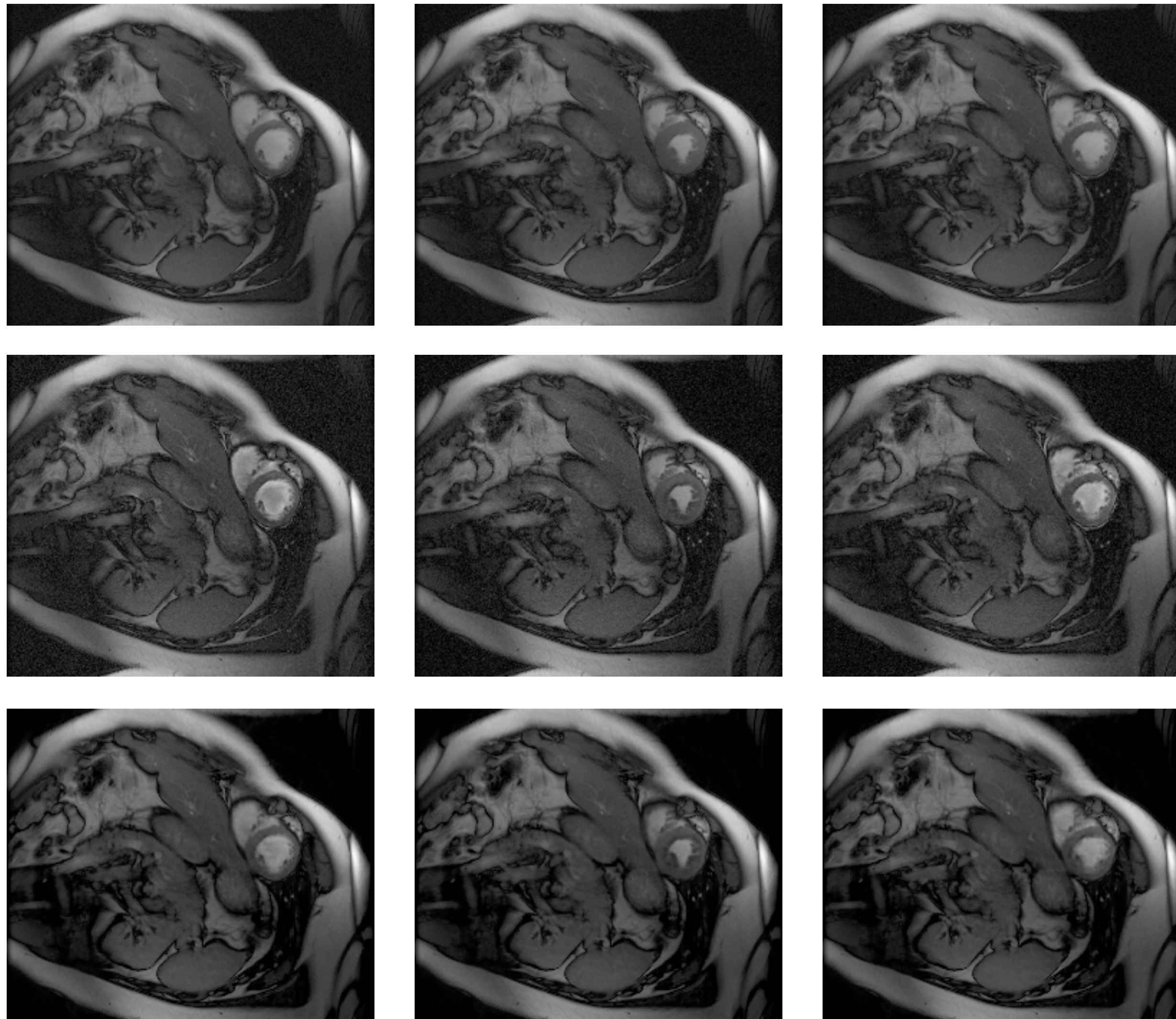
# Unknown sensitivity maps

**R=4**

**Fully-sampled (ground truth)**

**Without motion**

**With motion**



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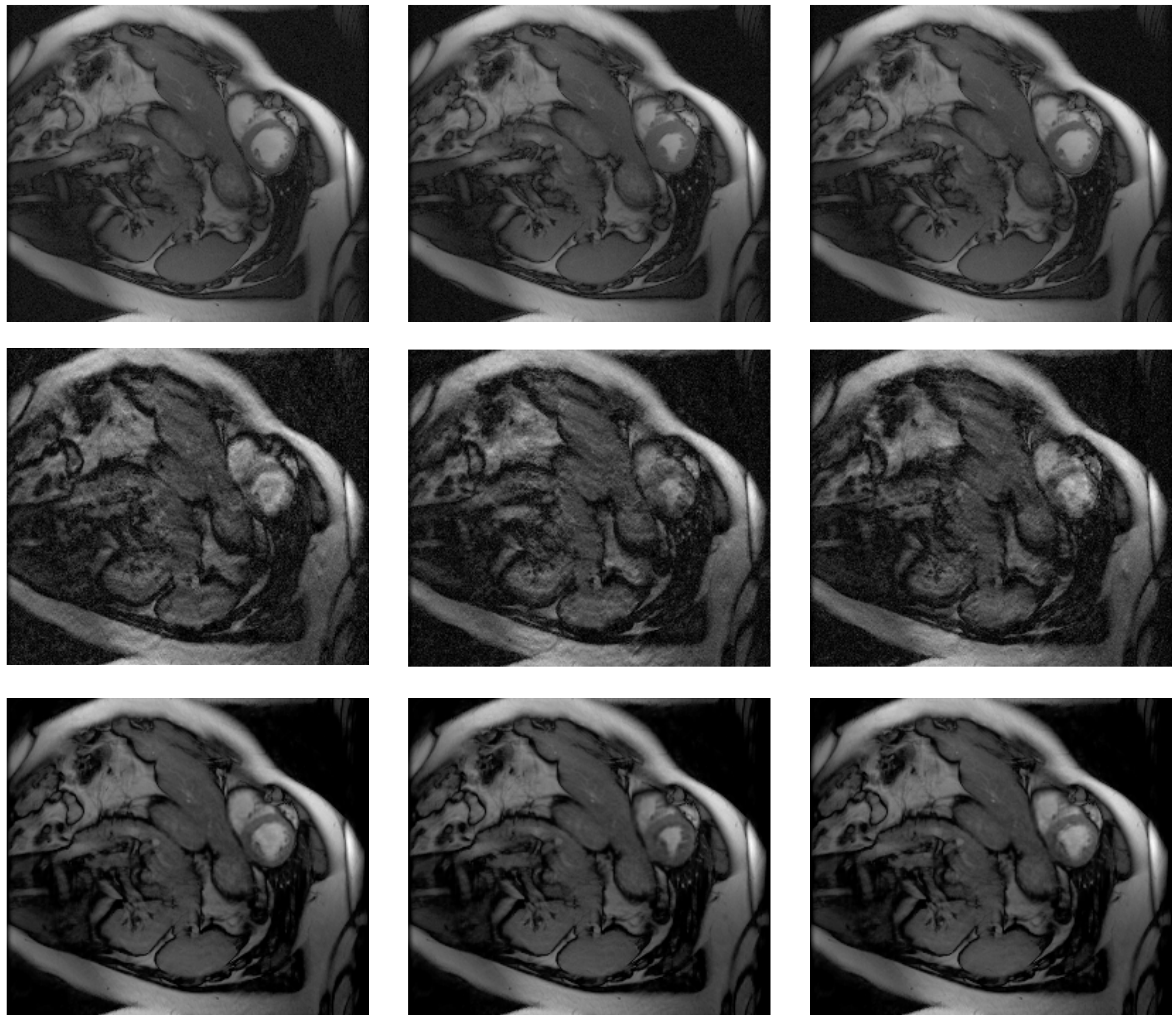
**Without motion**

**With motion**



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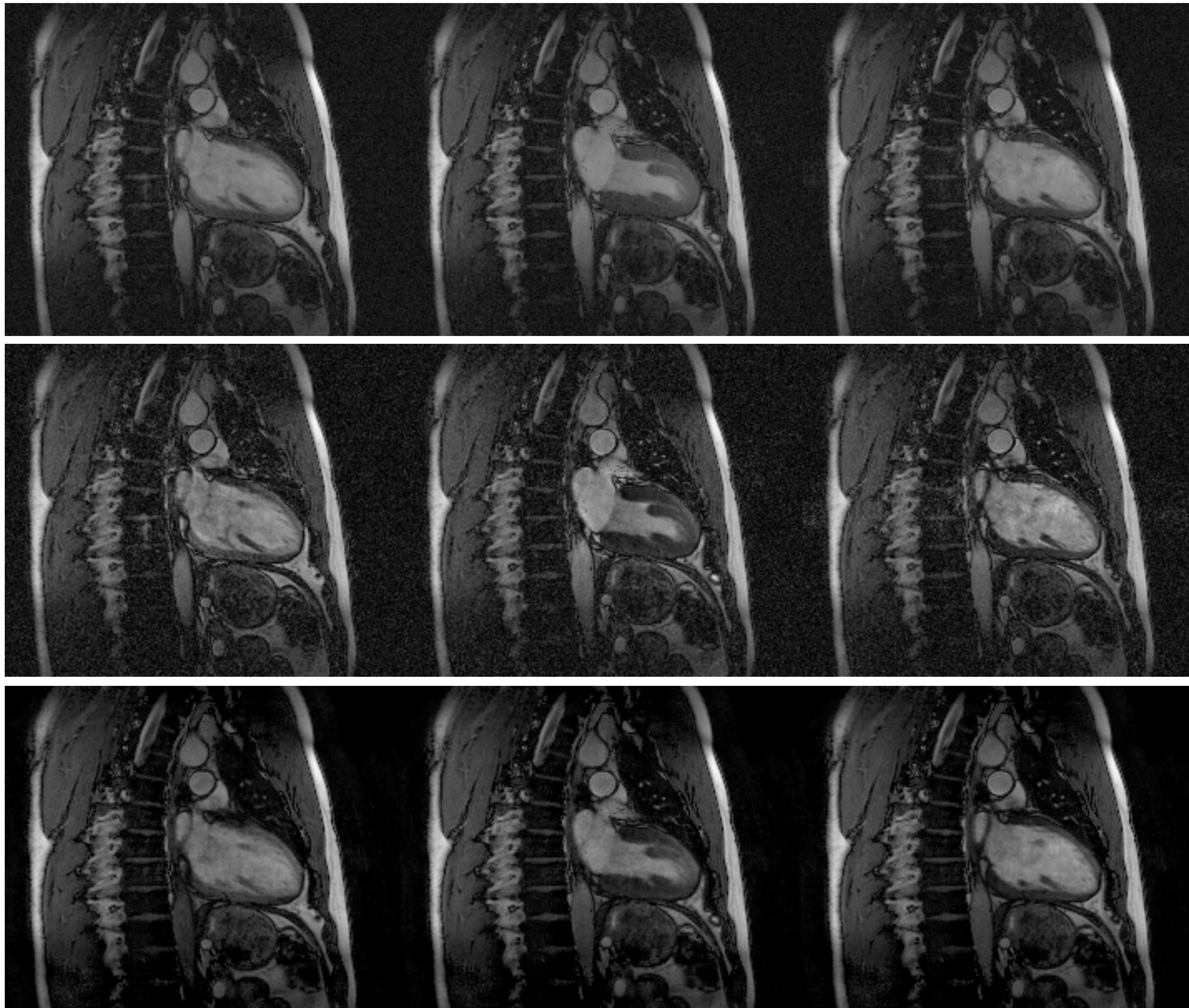

# Unknown sensitivity maps

**R=8**

**Fully-sampled (ground truth)**

**Without motion**

**With motion**