Communications

IB Paper 6

Handout 3: Digitisation and Digital Signals

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Lent Term

Outline

Typical Sources

Analogue Sources

Produce continuous outputs

- Speech
- Music
- (Moving/Static) images
- And also: temperature, speed, time...

using a device that converts the real signal to voltage.

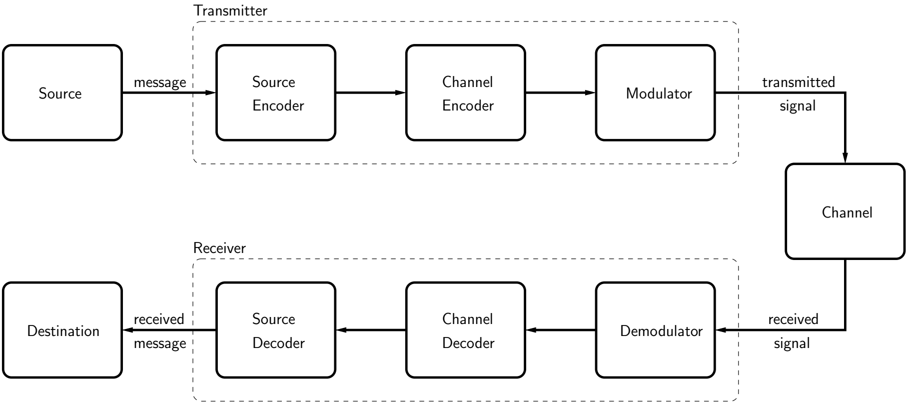
Typical Sources

Digital Sources

Produce digital outputs (binary, ASCII)

- Computer files
- E-mail
- Digital storage devices (CDs, DVDs)
- JPEG/MPEG files

Basic Block Diagram



Motivation

We need the ability to transform signals from analogue to digital (digitisation) or from digital to analogue (baseband modulation).

Digitisation of Analogue Signals

Digitisation

Is the process for which an analogue signal is converted into digital format, i.e., from a continuous signal (in time and amplitude) to a discrete signal (in time and amplitude). It consists of

- Sampling (discretises the time axis)
- Quantisation (discretises the signal amplitude axis)

Another possible name is analogue-to-digital conversion (ADC).

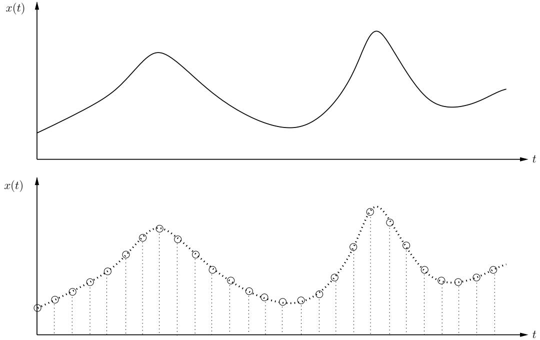
Sampling (recap)

What is sampling? (time domain)

Consider a signal $x(t)$ with bandwidth B. Then, the sampled version of $x(t)$ is

$$x_s(t) = x(t)\sum_n \delta(t - nT_s) = \sum_n x(nT_s)\delta(t - nT_s)$$

where T_s is the sampling period.



Sampling (recap)

What is sampling? (frequency domain)

The Fourier transform of a train of delta functions is a train of delta functions (indirectly stated in Handout 5 of Signals and Data Analysis, page 52 Haykin and Moher's book),

$$\sum_n \delta(t - nT_s) \longleftrightarrow \frac{1}{T_s} \sum_m \delta\left(f - \frac{m}{T_s}\right)$$

Then the Fourier transform of the sampled signal $x_s(t)$ is given by

$$X_s(f) = \mathcal{F}\left[x(t) \sum_n \delta(t - nT_s)\right] = \mathcal{F}[x(t)] * \mathcal{F}\left[\sum_n \delta(t - nT_s)\right]$$

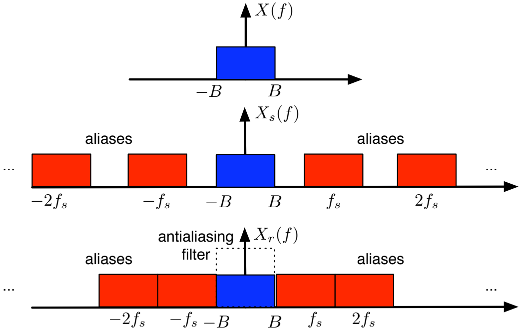
$$= X(f) * \frac{1}{T_s} \sum_m \delta\left(f - \frac{m}{T_s}\right) = \frac{1}{T_s} \sum_m X\left(f - \frac{m}{T_s}\right)$$

Sampling (recap)

What is sampling? (frequency domain)

Then the Fourier transform of the sampled signal $x_s(t)$ is given by

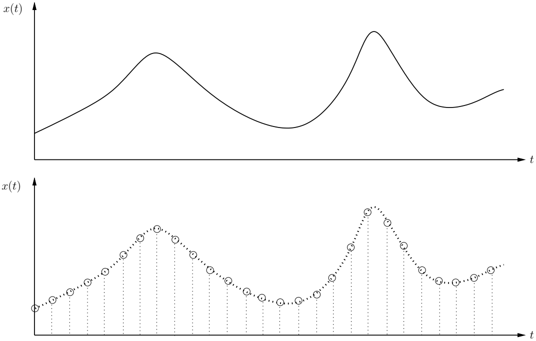
$$X_s(f) = \frac{1}{T_s} \sum_m X\left(f - \frac{m}{T_s}\right)$$



Sampling (recap)

Summarising: Nyquist Rate

Consider a signal $x(t)$ with bandwidth B. Then, we can recover $x(t)$ from its sampled version $x_s(T)$ provided that the sampling frequency is $f_s \geq 2B$ (using an ideal reconstruction or antialiasing filter).



Sampling (recap)

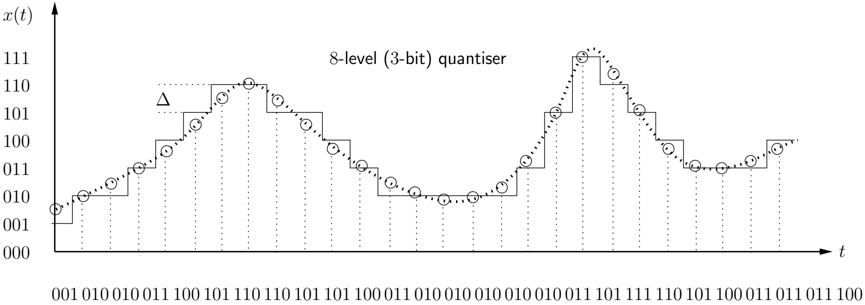
but...

This is now a discrete signal, not digital yet!

Quantisation

The Main Idea: Uniform Quantisation

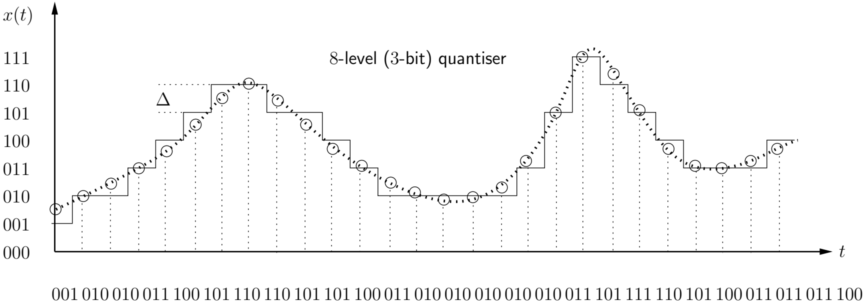
The sampled signal can take continuous values. To turn it into digital, we need to assign a discrete amplitude from a finite set of levels (with step Δ), and assign bits to those amplitudes.



Quantisation

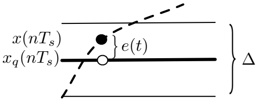
but...

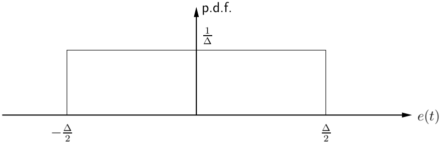
- Sampling is a reversible process (as long as we sample at least at the Nyquist rate)
- Quantisation is not! It introduces quantisation noise



Quantisation

The quantisation noise is $e(t) \triangleq x(t) - x_Q(t) \in \left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$, where $x_Q(t)$ is the quantised signal. We model $e(t)$ as a uniformly distributed random variable whose pdf is





we can easily compute the noise power

$$N_Q = \mathbb{E}[e^2] = \int_{-\Delta/2}^{\Delta/2} x^2 \frac{1}{\Delta} dx = \frac{1}{\Delta} \frac{x^3}{3} \Bigg|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{\Delta} \frac{(\Delta/2)^3}{3} - \frac{1}{\Delta} \frac{(-\Delta/2)^3}{3} = \frac{\Delta^2}{12}$$

and its corresponding RMS is $\frac{\Delta}{\sqrt{12}}$.

Quantisation

Signal-to-Noise Ratio

We can now compute the signal-to-noise ratio. Assume we have a sinusoidal signal taking values between $-V$ and $+V$ (in Volts). Since

$$\text{RMS signal} = \frac{V}{\sqrt{2}}$$

we have that for an n-bit (2^n level) quantiser $\Delta = 2V/2^n$ and hence

$$\text{SNR} = \frac{\text{signal power}}{\text{noise power}} = \frac{(\text{RMS signal})^2}{(\text{RMS noise})^2} = 3 \times 2^{2n-1}$$

$$= 1.76 + 6.02n \ \text{dB}$$

- Larger Δ,more quantisation noise (intuitive)
- More bits, larger SNR (better quality – intuitive), but more bits to be transmitted!!

Quantisation

Data Rate of the Quantised Source

Assuming we sample at Nyquist rate, and that we use an n-bit quantiser, the digitised source will have a rate of

$$R = n2B \text{ bits per second}$$

Example

Assume we want to digitise a speech signal, whose bandwidth $B = 3.2$kHz, using a Nyquist sampler and a 10-bit quantiser. What is the bit rate?

$$R = 10 \times 2 \times 3200 = 64000 \text{ bits per second} = 64 \text{ kbps}$$

A GSM phone uses a *clever* quantiser which reduces the bit-rate by a factor of 5, from 64kbps (our quantiser – Pulse Code Modulation (PCM)) to 13 kbps!!

Baseband modulation

So far...

- We have digital signals (strings of 0s and 1s)
  - Digitised (sampled and quantised) analog signals
  - Pure digital signals
- We need now to associate bits with signals
- Digital electronic devices operate with HIGH and LOW electrical states (voltage)

Baseband Modulation

Signal Representation

We represent digital signals as a pulse train

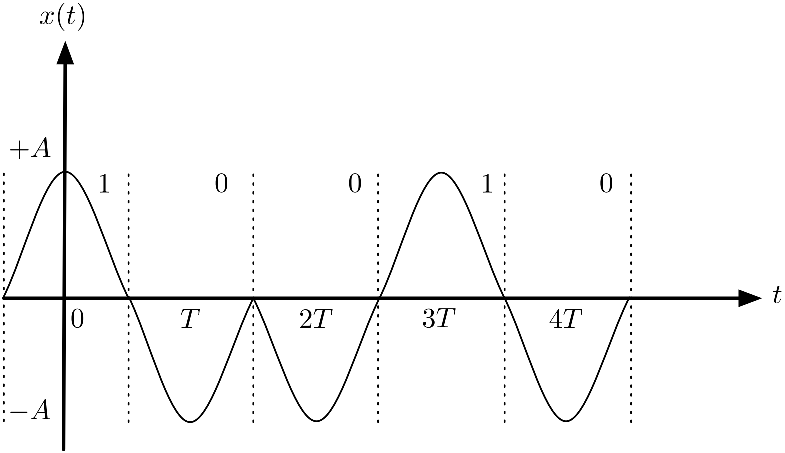
$$x(t) = \sum_k a_k p(t - kT)$$

- a_k is the k-th symbol in the message sequence
  - a_k could be just bits, 0s and 1s
  - a_k could belong to a set of M discrete values
- T is the symbol period
- $p(t)$ is the pulse such that

$$p(t) = \begin{cases} 1 & t = 0 \\ 0 & t = \pm T, \pm 2T, \ldots \end{cases}$$

- This is called pulse amplitude modulation (PAM) (no carrier modulation yet!)

Baseband Modulation



Baseband Modulation

What is the spectrum of the modulated signal?

Consider now, that a_k is a sequence of random symbols belonging to a certain alphabet. e.g., $\{-A, +A\}$ for binary PAM. Assuming that

- symbols have zero mean $\mathbb{E}[a_k] = 0$
- symbols are uncorrelated $\mathbb{E}[a_k a_j] = \delta_{kj}$, i.e., $\mathbb{E}[a_k a_j] = 1$ if $j = k$, and zero otherwise

the power spectral density of the pulse shaped digital signal is given by

$$|X(f)|^2 = \frac{1}{T}|P(f)|^2$$

Question

What is the spectrum (power spectral density) of a binary digital signal using triangular pulses? (a) a delta, (b) sinc^2, (c) sinc^4, (d) what?

Baseband Modulation

Concept of Rate

How fast can information be transmitted?

$$R = \frac{1}{T} \qquad \text{in symbols per second, or baud}$$

$$R_b = \frac{\log_2 M}{T} \qquad \text{in bits per second}$$

Main goal of Communications...

... to reliably transmit the largest possible data rate (in bits/second).