

# PSD of $\cos(\omega_0 t)$

$$x_c(t) = \begin{cases} \cos(\omega_0 t) & t \in [-T/2, T/2] \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} X_c(\omega) &= \int_{-T/2}^{+T/2} \cos(\omega_0 t) e^{-j\omega t} dt \\ &= \frac{1}{2} \int_{-T/2}^{+T/2} e^{-j(\omega - \omega_0)t} dt + \frac{1}{2} \int_{-T/2}^{+T/2} e^{-j(\omega + \omega_0)t} dt \\ &= \frac{\sin(\omega - \omega_0)T/2}{\omega - \omega_0} + \frac{\sin(\omega + \omega_0)T/2}{\omega + \omega_0} \end{aligned}$$

if  $\omega \neq \omega_0$  and  $-\omega \neq \omega_0$ , then  $X_c(\omega)$  is a finite number and  $\lim_{T \rightarrow \infty} \frac{1}{T} |X_c(\omega)|^2 = 0$

Since the power is

$$\begin{aligned} P_x &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |\cos(\omega_0 t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \left( \frac{1}{2} + \frac{1}{2} \cos(2\omega_0 t) \right) dt \\ &= \frac{1}{2} + \lim_{T \rightarrow \infty} \frac{1}{T} \frac{1}{2\omega_0} \sin 2\omega_0 t \Big|_{-T/2}^{T/2} = \frac{1}{2} \end{aligned}$$

hence PSD

