Part IB Paper 6: Information Engineering

COMMUNICATIONS

Examples Paper 6/9: Channels, Digital Modulation and Multiple Access

1. An analog signal of bandwidth $B_x = 5$kHz is digitised using a uniform 8-bit quantiser.

   (a) Calculate the minimum data rate of the digitised source.

   (b) A part of the output bit sequence is 0 1 1 0 1 0 1. If we use the bit-to-voltage association $0 \to a = 0$V and $1 \to a = +A$V, sketch the corresponding ASK modulated signal if we use a rectangular pulse.

   (c) If we now associate $0 \to a = -A$V and $1 \to a = +A$V, sketch the corresponding ASK modulated signal and the corresponding spectrum if we now use a triangular pulse.

   (d) Show that under the same bit-to-voltage association of question (c), the ASK and PSK modulated signals are the same when a rectangular pulse is used.

   (e) Calculate the error probability using the assignment $0 \to a = 0$V and $1 \to a = +A$V and compare to that of BPSK given in the lecture notes. Which one is smaller? Why?

2. A cable has a first-order low-pass frequency response

$$H(j\omega) = \frac{0.1}{1 + j\omega\tau}$$

   and hence impulse response

$$h(t) = \begin{cases} \frac{0.1}{\tau} e^{-\frac{t}{\tau}} & t > 0 \\ 0 & t \leq 0. \end{cases}$$

   Binary signals are to be transmitted over the cable at a rate of $R = \frac{1}{T}$ bit/s, using rectangular pulses of duration T seconds and amplitude $+A$ V to transmit a 1 and 0 V to transmit a 0.

   (a) Compute the step response, namely, the output voltage V_1 when the input is a long run of data 1s.

   (b) Assume that for satisfactory operation it is necessary, when a single 1 pulse is input after a long run of 0s, for the output to reach at least 80% of V_1. By computing the cable output when a single 1 pulse is input, determine the maximum allowable transmission rate R.

3. The capacity (in bits/channel use) of the binary symmetric channel (BSC) is

$$C = 1 - h(\epsilon) \qquad \text{bits/channel use}$$

where

$$h(\epsilon) = -\epsilon \log_2(\epsilon) - (1-\epsilon)\log_2(1-\epsilon)$$

is the binary entropy function.

(a) Describe the meaning of channel capacity.

(b) Based on the plot of the capacity of the BSC from the lecture notes, can a rate of $R = 0.4$ bits/channel use be transmitted reliably over a BSC with crossover probability $\epsilon = 0.3$? Why?

(c) If every channel use corresponds to 0.1 ms, calculate the maximum channel transition probability $\epsilon^\star$ to reliably support a data rate of 3kbit/s. Use the table below for the function $h^{-1}(x)$.

(d) Calculate the bit error probability of a repetition code of rate $R = \frac{1}{5}$ transmitted over a binary symmetric channel (BSC) with crossover probability ϵ.

(e) Why are repetition codes not effective to approach the channel capacity?

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$h^{-1}(x)$	0	0.013	0.031	0.053	0.079	0.11	0.146	0.189	0.243	0.316	0.5

4. (a) What is the minimum signal-to-noise ratio (in dB) for reliable communication over an additive white Gaussian channel with bandwidth $B = 200$kHz, if we want to transmit a data rate of 64kbit/s?

(b) What is the spectral efficiency?

(c) What is the spectral efficiency if we add a channel code of rate $R_c = 1/2$ to the transmission chain?

5. Consider a multiple-access channel with K users and a total bandwidth B.

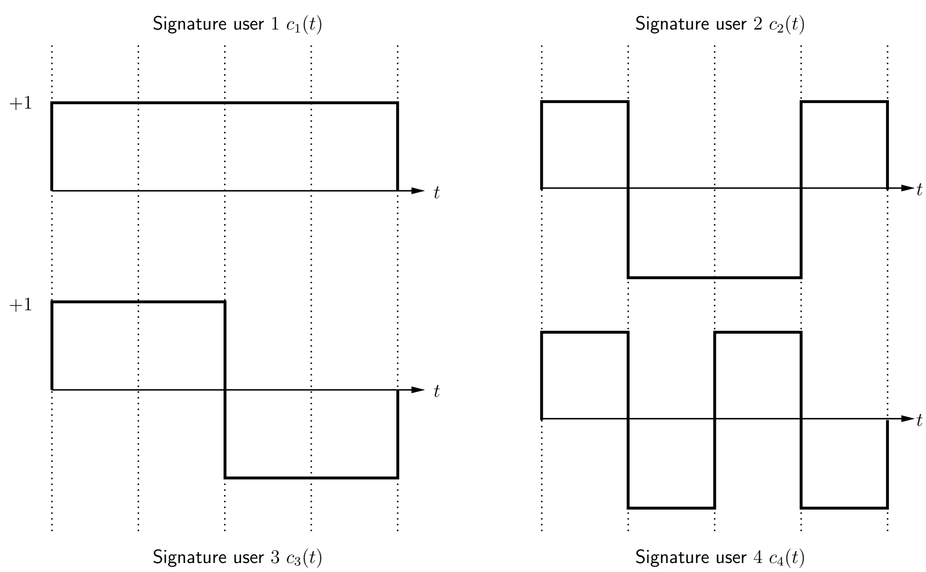
   (a) Explain how FDMA, TDMA and CDMA work, and outline the main differences between the three.

   (b) How many users can be accommodated in an FDMA system with total bandwidth 20MHz, if each user employs binary PSK modulation with rectangular pulses at a rate of $R = 200$kbit/s? (suppose that the spectrum of the BPSK signal does not cause interference beyond the first side lobe).

   (c) What is the capacity per user of an FDMA system? Sketch the spectral efficiency per user $c_k = \frac{C_k}{B_u}$ as a function of the signal-to-noise ratio per user $\mathrm{SNR}_u = \frac{P}{N_0 B_u}$. What is the total sum-capacity, defined as the sum of all the user capacities?

   (d) What is the capacity of TDMA if users can use a maximum power P every time they transmit (this is a different scenario than the one in the notes)? Which system (FDMA or TDMA) has larger capacity per user?

   (e) Show that the signature signals in the Figure are orthogonal in a CDMA system with $K = 4$ users.



Answers:

1. a) 80kbit/s.

2. a) $0.1 \times A$, b) $0.62/\tau$.

3. b) not possible; c) $\epsilon^\star = 0.189$.

4. a) 6.05dB, b) 0.32, b) 0.16.

5. b) 25 users.